An evolving network model with information filtering and mixed attachment mechanisms

Xikun Huang[a,b,*], Ruqian Lu[a]

[a] *Academy of Mathematics and Systems Science & Key Lab-MADIS Chinese Academy of Sciences, Beijing 100190, China*
[b] *School of the Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China*

Abstract

In this paper, we propose an evolving network model with information filtering and mixed attachment mechanisms. We theoretically analyze the in-degree distribution of networks generated by the proposed model in two special cases. And we prove that if the new coming vertex does not filter information, then the in-degree distribution is power-law, and the power-law exponent has range $(3/2, \infty)$. Otherwise, the in-degree distribution becomes complex which has a transition between exponential and power-law scaling. Numerical simulations are consistent with analytical results. In addition, we calculate various measures of networks generated by the proposed model, and compare values of these measures with that of networks generated by uniform attachment model, Barabási-Albert model and copying model. It shows that our model can generate networks with more diverse topological features.

Keywords: Network generative model, Information filtering, Preferential attachment, Copying, In-degree distribution

1. Introduction

Modeling complex networks have been in the forefront of network science research for almost two decades. Empirical studies on real-life networks such as social networks, biological networks , citations networks and the World Wide Web have shown that many networks exhibit common topological features[1]-[6]. In particular, two important characteristics that many real networks share are small-world behavior [7] and power-law degree distribution [8].

A great deal of network models have been formulated to capture the properties of real networks [8]-[16]. One of the earliest models is the classical Erdős-Rényi(ER) random graph model [9], where edges are distributed randomly among a fixed number of vertices. However, the degree distribution of

*Corresponding author
Email address: huangxikun14@mails.ucas.ac.cn (Xikun Huang)

the network generated by the ER model is Poisson distribution which is not consistent with that of many real networks. In 1999, Barabási and Albert [8] proposed a growing network model, also called BA model, which can generate networks with power-law degree distribution. The BA model has two important ingredients: growth and preferential attachment. Growth means that network continuously expands by the addition of new vertices with several edges attached to it, and preferential attachment means that new added vertex tends to connect to high-degree vertices rather than low-degree vertices [17].

Despite its great success in explaining power-law degree distribution of real networks, the BA model possesses several limitations. Actually, there is an implicit assumption in the BA model that the new added vertex knows the degree values of all existing vertices, and this assumption does not hold in many real networks. For example, in social networks, it is impossible for a person to know all other people's information when he or she makes friends in a new city or a new school. And it is more reasonable to assume that the newly coming vertex only owns partial information of the entire network or it filters information when it chooses targets. Several models have tried to describe this phenomenon [18][10][19][13]. Mossa et al. [18] proposed a model which incorporates information filtering, and they found that the in-degree distribution decays as a power law with an exponential truncation. Li and Chen [10] proposed a local-world evolving network model, where the new vertex will firstly select M_t vertices randomly from the existing network as its "local world", and then it only connects to m vertices that are all selected from its local world with preferential attachment rule. Li et al. showed that the degree distribution of the network generated by their proposed model represents a transition between exponential and power law. The above-mentioned models successfully incorporate information filtering or local world effect, but they have ignored the fact that the new vertex may connect to vertices that are not in its local world even the new vertex does not own the information of those vertices. In a social network for instance, it is common that our friends will introduce their friends, who are strangers for us, to us. Similar phenomenon can also be found in citation networks where researchers may refer those papers that appeared in papers they have read. We can mimic this behavior by using copying mechanism [20]. Kleinberg et al. [20] proposed a model which integrates copying mechanism when they studied the World Wide Web, where a new vertex chooses an old vertex as its prototype vertex and copies some out-neighbors of this prototype vertex to be its own out-neighbors.

In this paper, inspired by above thoughts, we propose an evolving network model with information filtering and mixed attachment mechanisms, and we call it the IFMAM model. Information filtering means that the new vertex will firstly choose some vertices randomly from the existing network as its local world. And then the new vertex can connect to vertices chosen from its local world with preferential attachment rule, or it can copy behaviors of an existing vertex which is chosen randomly from its local world. As a result, on the one hand, the new vertex filters information by choosing its local world and it knows the complete information of the chosen local world. On the other hand, it is possible for the new vertex to connect to any vertex of the existing network, even those that are not in its local word. Several models such as uniform attachment model [17] and the BA model [8] are limiting cases of the proposed model. If the new vertex select all existing vertices as its local world, we prove that the in-degree distribution is a power-law distribution, and the power-law exponent has range $(3/2, \infty)$. If the new vertex select a fixed number of existing vertices as its local world, we also derive the asymptotic solution of the in-degree distribution, which is complex and has a transition between exponential and power-law scaling. Additionally, we numerically explore properties of networks generated by the IFMAM model, such as clustering coefficient, average path length and so on. It shows that the proposed model can generate networks with small-world behavior. We also compare the values of these measures with that of networks generated by uniform attachment model, the BA model and the copying model, and it shows that the IFMAM model can generate networks with more diverse structural properties.

The rest of the paper is organized as follows: The IFMAM model is introduced and analyzed in Section 2. Properties of complex networks generated by the IFMAM model are analyzed in Section 3. Finally, we conclude in Section 4.

Notation	Meaning
G_t	Graph at time t
V_t	Vertex set at time t
N_t	Number of vertex at time t
I_v^t	In-degree of vertex v at time t
D_v	Degree of vertex v
α	The exponent of power-law distribution $p(x) \sim x^{-\alpha}$
m	Number of edges added at each time step
γ	Copy factor
C_t	Vertex set of the local world of the vertex v_t
M_t	Size of C_t; i.e. $M_t = \|C_t\|$
M	Size of C_t when M_t is a constant
a	Initial attractiveness of each vertex
$\Pi_t(v)$	The probability that the vertex v in C_t is chosen by preferential attachment rule at time t.
$p_k(N_t)$	The proportion of vertices whose in-degree are k when the size of graph is N_t

Table 1: Notations used in this paper

2. The Model

In this section, we firstly introduce the IFMAM model which considers information filtering and combines preferential attachment and copying mechanisms. And then we analyze the in-degree distribution of networks generated by the proposed model. Before going into further detail, we give notations used in this paper in Table 1.

2.1. Model Description

We consider directed graph model and assume that all vertices have same out-degree. If there is an edge originating from vertex u and ends at vertex v, then we call v is an *out-neighbor* of u, so each vertex has exactly m out-neighbors. The model is described by a graph process $\{G_t\}_{t\geq 0}$. Start with a directed graph G_0 consisting of m_0 vertices where every vertex connects to other $m < m_0$ vertices randomly. For $t \geq 1$, the graph G_t is constructed from G_{t-1} by following steps:

1. Add a new vertex v_t and m new edges originating from v_t, into G_{t-1}. And the other ends of new edges are chosen by following rules described in step 2 and 3.
2. Choose a non-empty subset of V_{t-1} as the local world of v_t, using C_t to denote this subset. Choose a vertex u_t from C_t randomly as the prototype vertex of v_t.
3. Consider u_t's out-neighbors one by one, with probability γ, let v_t connect to this out-neighbor, and with probability $1-\gamma$, let v_t connect to a vertex p in C_t selected by preferential attachment rule; i.e. $\Pi_t(p) = \frac{I_p^{t-1}+a}{\sum_{q\in C_t}(I_q^{t-1}+a)}$, where a is a constant which represents initial attractiveness of each vertex.

It is worthy to note that several previous models are limiting cases of undirected versions of the proposed model. We use M_t to represent the size of v_t's local world. The proposed model becomes the BA model[8] when $\gamma = 0, a = m, M_t = N_t$. When $\gamma = 0, a = m, M_t = m$, our model is same as model A in [17], where the preferential attachment rule in BA is replaced by uniform attachment, and we call it uniform attachment model. And when $M_t = m$ for all t, the preferential attachment selection in C_t will be equivalent to random selection from V_t to some extent, because vertices in C_t are chosen randomly from V_t and the out-degree of the new vertex is m. In this case, the IFMAM model is similar to copying model.

2.2. In-degree Distribution

In this subsection, we use the method in Newman's book [5] to derive the in-degree distribution of the network generated by the IFMAM model. Let N_t denote the size of G_t and I_v^t denote the in-degree of vertex v at time t, then at time $t+1$, an old vertex v may be chosen as the new coming vertex v_{t+1}'s out-neighbor by two possible ways. One is that vertex v is an out-neighbor of v_{t+1}'s prototype vertex and is chosen with probability $\frac{\gamma I_v^t}{N_t}$. The other is that v is in the local world of v_{t+1} and is chosen with preferential attachment rule. So, for vertex v, the expectation of gotten new edges at time $t+1$ is

$$\frac{\gamma I_v^t}{N_t} + (1-\gamma)m\frac{M_t}{N_t}\frac{I_v^t + a}{\sum_{u\in C_t}(I_u^t + a)} \tag{1}$$

Below we discuss the in-degree distribution of the network in two cases. One is that the new vertex selects all existing vertices as its local world; i.e. $M_t = N_t$. The other is that the size of local world of all vertices are same; i.e. $M_t = M$, where M is a constant.

2.2.1. Case A: $M_t = N_t$

If $M_t = N_t$, equation (1) becomes

$$\frac{\gamma I_v^t}{N_t} + (1-\gamma)m\frac{I_v^t + a}{N_t(m+a)}$$

When the size of graph is N_t, for all vertices whose in-degree are k, the expectation of gotten new edges is

$$N_t \times p_k(N_t) \times [\frac{\gamma k}{N_t} + \frac{(1-\gamma)m}{N_t}\frac{k+a}{m+a}] = [\gamma k + (1-\gamma)m\frac{k+a}{m+a}]p_k(N_t) \quad (2)$$

where $p_k(N_t)$ is the proportion of vertices whose in-degree are k when the size of graph is N_t.

And the master equation for in-degree distribution is

$$\begin{aligned}(N_t+1)p_k(N_t+1) = {} & N_t p_k(N_t) \\ & + [\gamma(k-1) + (1-\gamma)m\frac{k-1+a}{m+a}]p_{k-1}(N_t) \\ & - [\gamma k + (1-\gamma)m\frac{k+a}{m+a}]p_k(N_t)\end{aligned} \quad (3)$$

This equation is true except for $k = 0$. When $k = 0$ the second term of the right part of the equation disappears and the new added vertex's in-degree is zero. So the equation becomes

$$(N_t+1)p_0(N_t+1) = N_t p_0(N_t) + 1 - \frac{(1-\gamma)ma}{m+a}p_0(N_t) \quad (4)$$

Let $t \to \infty$ and $p_k \equiv p_k(\infty)$ we have

$$p_k = [\gamma(k-1) + (1-\gamma)m\frac{k-1+a}{m+a}]p_{k-1} - [\gamma k + (1-\gamma)m\frac{k+a}{m+a}]p_k \quad (5)$$

$$p_0 = 1 - \frac{(1-\gamma)ma}{m+a}p_0 \quad (6)$$

where $k \geq 1$.

Rearrange above equations we have

$$p_k = \frac{(m+a)\gamma(k-1) + (1-\gamma)m(k-1+a)}{(m+a)(1+\gamma k) + (1-\gamma)m(k+a)}p_{k-1} \quad (7)$$

$$p_0 = \frac{m+a}{m+a+(1-\gamma)ma} \tag{8}$$

The asymptotic solution of equation (7) and (8) is

$$p_k = \frac{1}{k}\frac{m+a}{m+a\gamma}\frac{B(k+1,\frac{m+a+(1-\gamma)ma}{m+a\gamma})}{B(k,1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma})} \tag{9}$$

where $B(x,y)$ is Beta function. The derivation details are given in Appendix A. Using the property of Beta function

$$B(x,y) \approx x^{-y}\Gamma(y) \quad for \quad x \to \infty \quad and \quad x \gg y$$

When k is large enough, the in-degree distribution behaves like

$$\begin{aligned} p_k &\sim k^{-1}\frac{(k+1)^{-[\frac{m+a+(1-\gamma)ma}{m+a\gamma}]}}{k^{-[1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma}]}} \\ &\sim k^{-\alpha} \end{aligned} \tag{10}$$

where

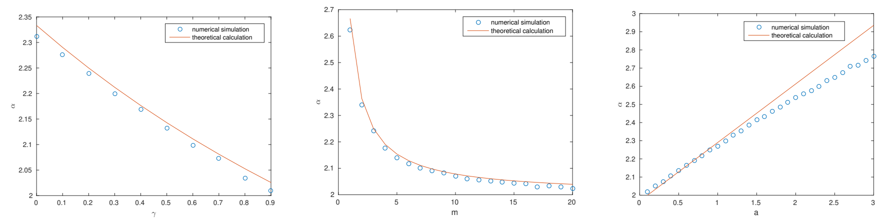
$$\alpha = 1 + \frac{m+a}{m+\gamma} \tag{11}$$

So we get that when $M_t = N_t$, the in-degree distribution has a power-law form and the power law exponent is $1 + \frac{m+a}{m+\gamma}$, which has range $(3/2, \infty)$. Note that we have mentioned in Section 2.1 that the undirected version of the proposed model is same as the BA model when $\gamma = 0, a = m$ and $M_t = N_t$. In this case, our derivation is consistent with that of the BA model, i.e. $\alpha = 1 + \frac{m+m}{m+0} = 3$. We also carry out numerical simulations to verify our derivation. In Fig 1, we show the relationship between the power-law exponent and parameters of the IFMAM model. We use the method proposed by Clauset et al. [21] to compute the power-law exponent. We can see that numerical results are coincident with theoretical results.

2.2.2. Case B: M_t is a fixed constant

Now we consider the situation that M_t is a fixed constant M and $a = 1$. When the size of graph is N_t, for all vertices whose in-degree are k, the expectation of gotten new edges is

$$\begin{aligned} &N_t \times p_k(N_t) \times [\frac{\gamma k}{N_t} + (1-\gamma)m\frac{M}{N_t}\frac{k+1}{\sum_{u\in C_t}(I_u^t+1)}] \\ &= [\gamma k + (1-\gamma)mM\frac{k+1}{\sum_{u\in C_t}(I_u^t+1)}]p_k(N_t) \end{aligned} \tag{12}$$



(a) power-law exponent versus copy factor (b) power-law exponent versus out-degree (c) power-law exponent versus initial attractiveness

Figure 1: Comparison between the analytical results (lines) derived from Eq.(11) and the simulation results (circles) for different parameter values. The quantity shown is the exponent α of the power law in-degree distribution of the resulting network. (a) $N = 10000000$, $m = 3$ and $a = 1$. (b) $N = 10000000$, $\gamma = 0.2$ and $a = 1$. (c) $N = 10000000$, $\gamma = 0.1$ and $m = 3$.

Then we can get the master equation. For $k \geq 1$

$$\begin{aligned}(N_t+1)p_k(N_t+1) &= N_t p_k(N_t) \\ &+ [\gamma(k-1) + (1-\gamma)mM\frac{k}{\sum_{u\in C_t}(I_u^t+1)}]p_{k-1}(N_t) \\ &- [\gamma k + (1-\gamma)mM\frac{k+1}{\sum_{u\in C_t}(I_u^t+1)}]p_k(N_t)\end{aligned} \tag{13}$$

And for $k = 0$

$$\begin{aligned}(N_t+1)p_0(N_t+1) &= N_t p_0(N_t) + 1 \\ &- [(1-\gamma)mM\frac{1}{\sum_{u\in C_t}(I_u^t+1)}]p_0(N_t)\end{aligned} \tag{14}$$

It is worthy to note that we cannot get the exact value of the sum term $\sum_{u\in C_t}(I_u^t+1)$. By using similar tricks in [13], we can approximate the sum term. When we know C_t contains a vertex whose in-degree is k, we can approximate the sum term as $\sum_{u\in C_t}(I_u^t+1) = k + (M-1)m + M$. Substitute approximations into equation (13) and (14) , for $q \geq 1$ we have

$$\begin{aligned}(N_t+1)p_k(N_t+1) &= N_t p_k(N_t) \\ &+ [\gamma(k-1) + (1-\gamma)mM\frac{k}{k-1+(M-1)m+M}]p_{k-1}(N_t) \\ &- [\gamma k + (1-\gamma)mM\frac{k+1}{k+(M-1)m+M}]p_k(N_t)\end{aligned} \tag{15}$$

And for $k = 0$ we have

$$\begin{aligned}(N_t+1)p_0(N_t+1) &= N_t p_0(N_t) + 1 \\ &- [(1-\gamma)mM\frac{1}{(M-1)m+M}]p_0(N_t)\end{aligned} \tag{16}$$

Let $t \to \infty$ and $p_k \equiv p_k(\infty)$, then

$$\begin{aligned} p_k = & [\gamma(k-1) + (1-\gamma)mM\frac{k}{k-1+(M-1)m+M}]p_{k-1} \\ & - [\gamma k + (1-\gamma)mM\frac{k+1}{k+(M-1)m+M}]p_k \end{aligned} \tag{17}$$

And for $k = 0$

$$p_0 = 1 - [(1-\gamma)mM\frac{1}{(M-1)m+M}]p_0 \tag{18}$$

Rearrange above equations we get

$$p_k = \frac{\gamma(k-1) + (1-\gamma)mM\frac{k}{k-1+(M-1)m+M}}{1 + \gamma k + (1-\gamma)mM\frac{k+1}{k+(M-1)m+M}}p_{k-1} \tag{19}$$

$$p_0 = \frac{1+(M-1)m+M}{(1-\gamma)mM} \tag{20}$$

The asymptotic solution of equation (19) and (20) is complex, and we can only write it using Beta function as equation (21). The derivation details are given in Appendix B. The asymptotic solution is

$$\begin{aligned} p_k = & \frac{B(k, \frac{Mm+M\gamma-m\gamma+\sqrt{\nabla}+1}{2\gamma}+1)B(k, \frac{Mm+M\gamma-m\gamma-\sqrt{\nabla}+1}{2\gamma}+1)}{B(k, \frac{Mm+M\gamma-m\gamma+\sqrt{\triangle}}{2\gamma}+1)B(k, \frac{Mm+M\gamma-m\gamma-\sqrt{\triangle}}{2\gamma}+1)} \\ & \times \frac{1+(M-1)m+M}{\gamma k(k+(M-1)m+M)+(1-\gamma)mM(k+1)}\frac{k+(M-1)m+M}{(M-1)m+M} \end{aligned} \tag{21}$$

where

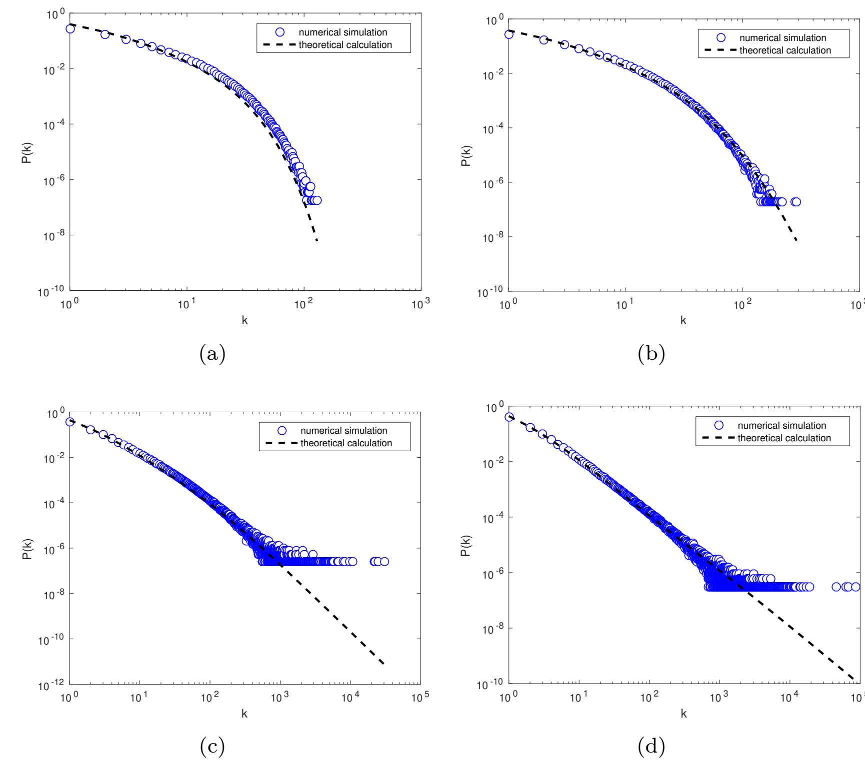
$$\triangle = M^2m^2 + 2M^2m\gamma + M^2\gamma^2 - 2Mm^2\gamma + 2Mm\gamma^2 - 4Mm\gamma + m^2\gamma^2$$

$$\nabla = M^2m^2 + 2M^2m\gamma + M^2\gamma^2 - 2Mm^2\gamma + 2Mm\gamma^2 - 8Mm\gamma + 2Mm - 2M\gamma + m^2\gamma^2 + 2m\gamma + 1$$

It is not easy to analyze the above solution completely. But when $k \gg \frac{Mm}{\gamma}$ we have

$$p_k \sim k^{-(1+\frac{1}{\gamma})}$$

And this is the same result as copying model [5]. In addition, we plot the in-degree distribution of networks generated by the IFMAM model with different M and γ values. And we compare the results obtained by theoretical calculation with numerical simulation in Fig 2. From Fig 2, we can see that when M and γ are both small, the in-degree distribution is similar to exponential distribution, and when M is getting larger, the in-degree distribution is similar to power-law distribution. And numerical simulations are consistent with theoretical results.



(a) (b) (c) (d)

Figure 2: Comparison between the analytical results (dashed lines) derived from Eq. (21) and the simulation results (circles) with different parameter values. The quantity shown is the in-degree distribution of the resulting network. And the values of input parameters are: (a) $N = 10000000, M = 3, m = 3, \gamma = 0.001$ (b) $N = 10000000, M = 3, m = 3, \gamma = 0.1$, (c) $N = 10000000, M = 10, m = 3, \gamma = 0.5$, (d) $N = 10000000, M = 100, m = 3, \gamma = 0.5$. The agreement is good.

3. Property of networks generated by the IFMAM Model

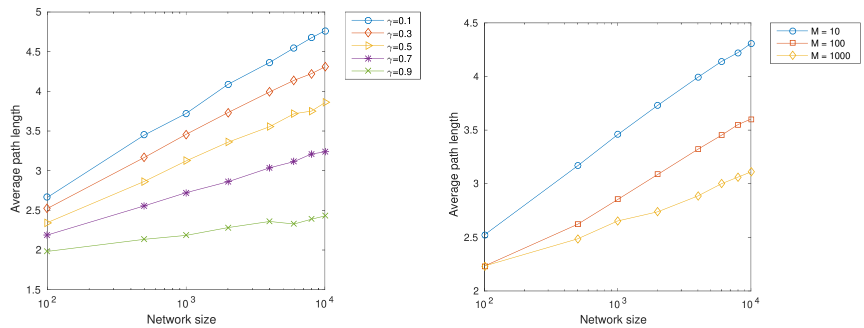
In this section, we numerically calculate several measures of networks generated by the IFMAM model. And the measures include average path length, clustering coefficient, number of triangles, effective diameter, spectral radius and assortativity. In addition, we compare values of these measures with that of networks generated by uniform attachment model, BA model and copying model.

We ignore the directionality of edges when calculating these measures using Python language package NetworkX [22]. And we focus on how these measures will change by varying the parameters γ and M.

3.1. Average path length

Average path length [23] is the average number of steps along the shortest paths for all possible pairs of network vertices. Most real networks have small average path length. In Fig 3, we plot average path length of networks generated by the IFMAM model with different parameter values. It shows that the average path length is negatively correlated to γ and M, and grows as $\ln N$ with network size N.

We also compare the average path length of networks generated by our model with that of networks generated by copying model [20], BA model [8] and uniform attachment model [17]. Fig5 shows that our model can generate networks which have almost the same average path length as copying model, BA model and uniform attachment model. On the other hand, the IFMAM model can generate networks with smaller average path length.



(a) (b)

Figure 3: The average path length versus network size with the given values of input parameters: (a) $M = 10$, $m = 3$, $a = 1$ and $\gamma = 0.1, 0.3, 0.5, 0.7$ and 0.9. (b) $M = 10, 100$ and 1000, $m = 3$, $a = 1$ and $\gamma = 0.3$.

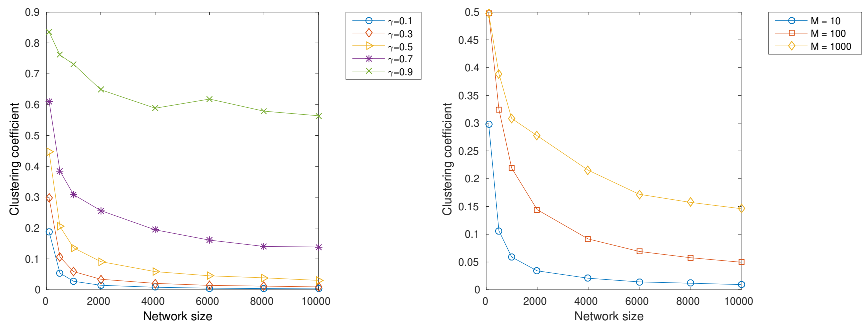
3.2. Clustering coefficient

The local clustering coefficient r_v for the vertex v is defined as

$$r_v = \frac{|\Gamma_v|}{1/2D_v(D_v - 1)}$$

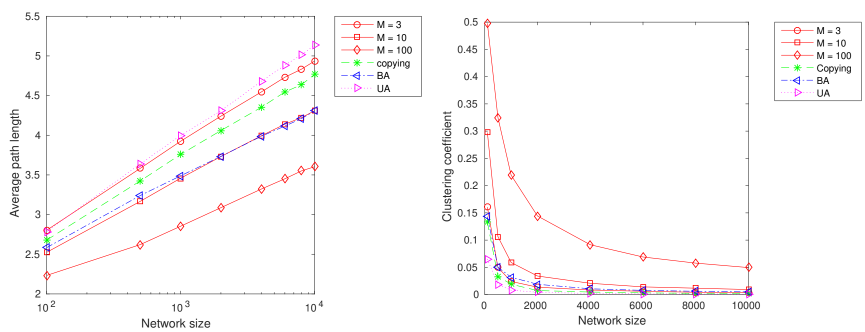
where $|\Gamma_v|$ is the number of actual edges existing in the network connecting v's neighbors [24]. The clustering coefficient of a network is defined as the average local clustering coefficient of all vertices in the network [7]. In Fig4, we plot clustering coefficient of networks generated by the IFMAM model with different parameter settings. It shows that the clustering coefficient is positively correlated to M and γ when the network size is fixed. As the network size increases, the clustering coefficient decreases firstly and then reaches a stable value gradually.

We also plot the clustering coefficient of networks generated by our model, copying model, BA model and uniform attachment model in Fig5. It shows that the IFMAM model can generate networks which has similar clustering coefficient as the three models. In addition, our model can generate networks with larger clustering coefficient.



(a) (b)

Figure 4: The clustering coefficient versus network size with the given values of input parameters: (a) $M = 10$, $m = 3$, $a = 1$ and $\gamma = 0.1, 0.3, 0.5, 0.7$ and 0.9. (b) $M = 10, 100$ and 1000, $m = 3$, $a = 1$ and $\gamma = 0.3$.



(a) (b)

Figure 5: Average path length and clustering coefficient of different size of networks generated by the IFMAM model, copying model, BA model and uniform attachment model(UA). We consider three parameter settings for the IFMAM model: $M = 3, 10$ and 100 when $m = 3$, $a = 1$, and $\gamma = 0.3$. We set copy factor $\gamma = 0.3$ for copying model, and out-degree $m = 3$ for uniform attachment model and BA model. (a) Average path length. (b) Clustering coefficient.

3.3. Other properties

The above results show that our model can indeed generate networks with small-world property; i.e. short average path length and high clustering coefficient [7]. Besides above mentioned properties, we also calculate other measures including number of triangles [14], effective diameter [4], spectral radius [25] and assortativity [26], which are plotted in Fig 6. And here we just explain the results briefly. Fig 6 shows that our model can generate diverse networks by tuning parameter γ and M. In addition, we compare values of these measures with that of networks generated by copying model, BA model and uniform attachment model in Fig7. It shows that the IFMAM model can generate networks which has similar triangle counts, effective diameter, and spectral radius as other three models. On the other hand, the IFMAM model can generate networks with larger triangle count and spectral radius, and smaller effective diameter. But it can only generate networks with disassortative mixing.

4. Conclusions

In this paper we have proposed an evolving network model, which we call the IFMAM model, that considers information filtering and mixed mechanisms. We have showed that several previous models are limiting cases of the proposed model. If the new vertex select all existing vertices as its local world, we prove that the in-degree distribution is a power-law distribution with power-law exponent $1 + \frac{m+a}{m+\gamma}$. If the new vertex select a fixed number of existing vertices as its local world, we also derive the asymptotic solution of the in-degree distribution, which is a little complex. Theoretical derivations were in good consistent with numerical simulations.

In order to investigate the properties of networks generated by the IFMAM model, we have calculated several network metrics including average path length, clustering coefficient, triangle count, effective diameter, spectral radius, and assortativity. We also compared the IFMAM model with previously mentioned models and showed that the IFMAM model are more powerful in the sense that it could generate networks with more diverge topological features.

Even though the proposed model are more powerful than previous models, it also has limitations. One of the limitations is that we just consider two cases of the size of new vertex's local world. In the future, we can consider models allowing different vertices may have different sizes of local world based on their capabilities. Furthermore, the future research can also focus on generating networks which is similar to given real-life networks, and we plan to solve this problem by parameter estimation in the future.

5. Acknowledgments

This work has been supported by National Key Research and Development Program of China under grant 2016YFB1000902, NSFC Project (Grant Nos. 61232015, 61472412, 61621003 , 61872352).

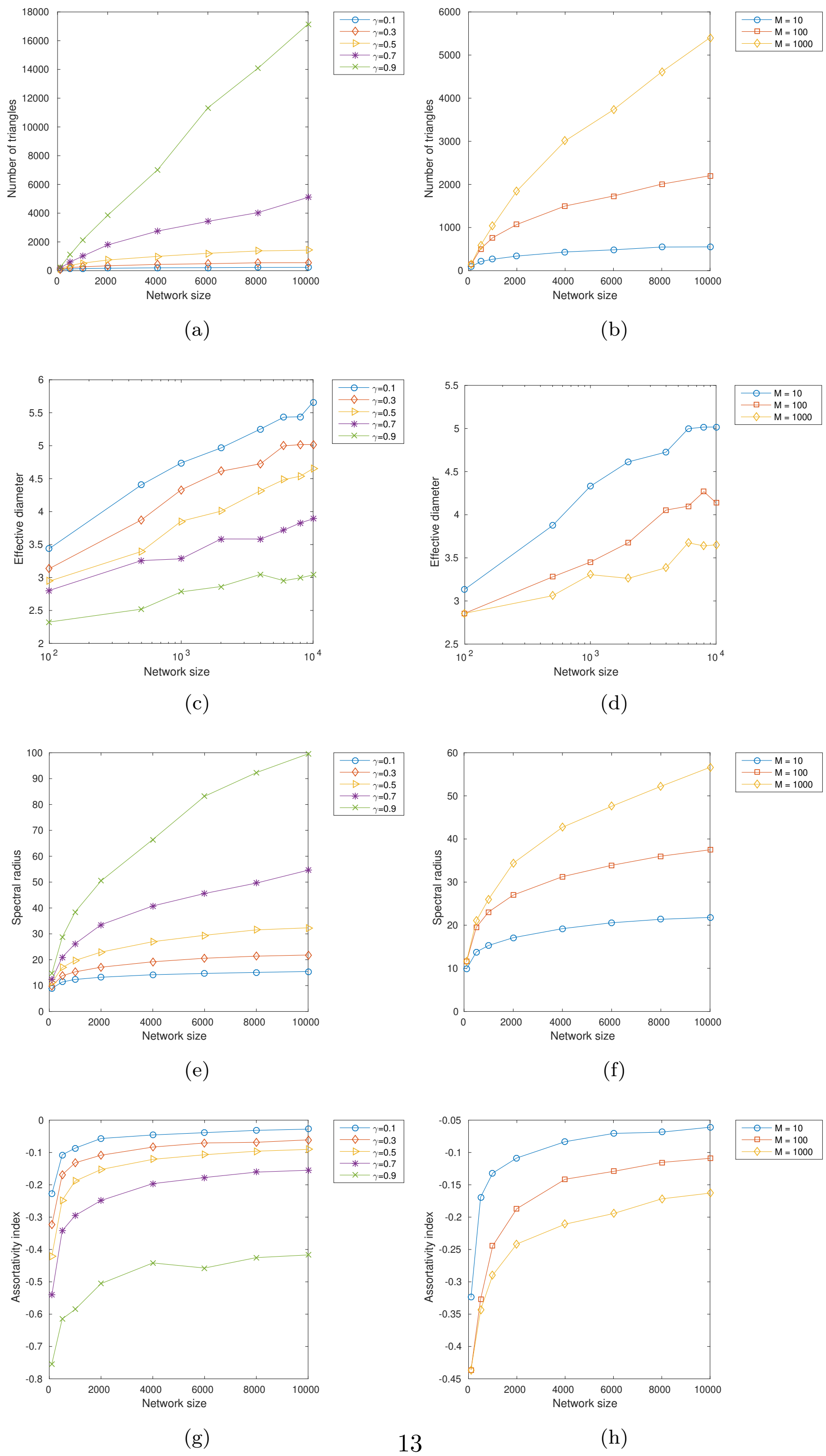
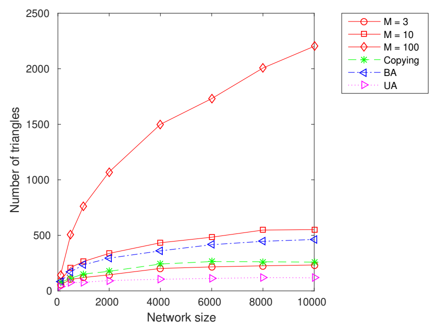
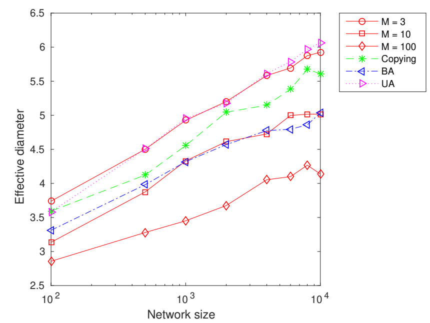


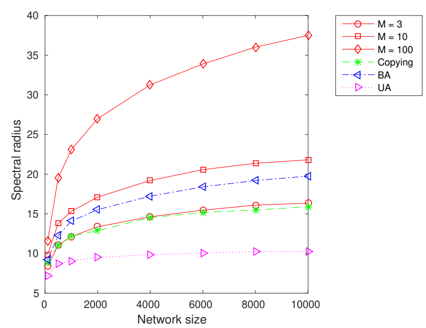
Figure 6: Various topological features versus network size with the given values of input parameters. (a)(c)(e)(g): $M = 10$, $m = 3$, $a = 1$ and $\gamma = 0.1, 0.3, 0.5, 0.7$ and 0.9. (b)(d)(f)(h): $M = 10, 100$ and 1000, $m = 3$, $a = 1$ and $\gamma = 0.3$.



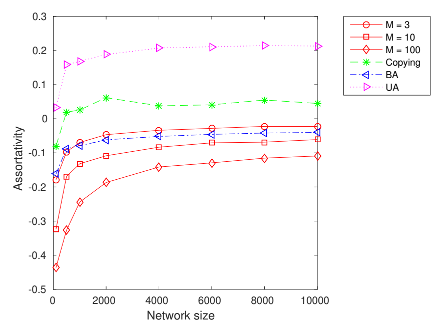
(a)



(b)



(c)



(d)

Figure 7: Various measures of different size of networks generated by the IFMAM model, copying model, BA model and uniform attachment model(UA). We consider three parameter settings for the IFMAM model: $M = 3, 10$ and 100 when $m = 3$, $a = 1$, and $\gamma = 0.3$. We set copy factor $\gamma = 0.3$ for copying model, and out-degree $m = 3$ for uniform attachment model and BA model. (a) Number of triangles. (b) Effective diameter. (c) Spectral radius. (d) Assortativity.

Appendices

A. Derivation of the in-degree distribution when $M_t = N_t$

In this section, we give the derivation of the asymptotic solution of the in-degree distribution when $M_t = N_t$. From equation (7) and (8), we have

$$\begin{aligned} p_k &= \frac{(m+a)\gamma(k-1)+m(1-\gamma)(k-1+a)}{(m+a)(1+\gamma k)+(1-\gamma)m(k+a)} p_{k-1} \\ &= \frac{(m+a\gamma)k+a(1-\gamma)(a-1)-(m+a)\gamma}{(m+a\gamma)k+m+a+(1-\gamma)ma} p_{k-1} \\ &= \frac{k+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma}}{k+\frac{m+a+(1-\gamma)ma}{m+a\gamma}} p_{k-1} \end{aligned} \tag{A.1}$$

And

$$\begin{aligned} p_0 &= \frac{m+a}{m+a+(1-\gamma)ma} \\ &= \frac{\frac{m+a}{m+a\gamma}}{\frac{m+a+(1-\gamma)ma}{m+a\gamma}} \end{aligned} \tag{A.2}$$

For general k , by iteration we have

$$\begin{aligned} p_k &= \frac{[k+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma}]\cdots[1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma}]}{[k+\frac{m+a+(1-\gamma)ma}{m+a\gamma}]\cdots[1+\frac{m+a+(1-\gamma)ma}{m+a\gamma}]} \frac{\frac{m+a}{m+a\gamma}}{\frac{m+a+(1-\gamma)ma}{m+a\gamma}} \\ &= \frac{m+a}{m+a\gamma} \frac{\Gamma(\frac{m+a+(1-\gamma)ma}{m+a\gamma})}{\Gamma(k+1+\frac{m+a+(1-\gamma)ma}{m+a\gamma})} \frac{\Gamma(k+1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma})}{\Gamma(1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma})} \\ &= \frac{1}{k}\frac{m+a}{m+a\gamma} \frac{\Gamma(\frac{m+a+(1-\gamma)ma}{m+a\gamma})\Gamma(k+1)}{\Gamma(k+1+\frac{m+a+(1-\gamma)ma}{m+a\gamma})} \frac{\Gamma(k+1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma})}{\Gamma(k)\Gamma(1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma})} \\ &= \frac{1}{k}\frac{m+a}{m+a\gamma} \frac{B(k+1,\frac{m+a+(1-\gamma)ma}{m+a\gamma})}{B(k,1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma})} \end{aligned} \tag{A.3}$$

When $x \to \infty$ and $x \gg y$, *Beta* function has the property

$$B(x,y) \approx x^{-y}\Gamma(y)$$

So when k is large enough, the in-degree distribution behaves like

$$\begin{aligned} p_k &\sim k^{-1} \frac{(k+1)^{-[\frac{m+a+(1-\gamma)ma}{m+a\gamma}]}}{k^{-[1+\frac{m(1-\gamma)(a-1)-(m+a)\gamma}{m+a\gamma}]}} \\ &\sim k^{-\alpha} \end{aligned} \tag{A.4}$$

where

$$\alpha = 1 + \frac{m+a}{m+\gamma} \tag{A.5}$$

B. Derivation of the in-degree distribution when $M_t = M$

In this section we try to find the asymptotic solution of equation (19) and (20).

$$p_k = \frac{\gamma(k-1)(k-1+(M-1)m+M)+(1-\gamma)mMk}{(1+\gamma k)(k+(M-1)m+M)+(1-\gamma)mM(k+1)} \frac{k+(M-1)m+M}{k-1+(M-1)m+M} p_{k-1} \tag{B.1}$$

We try to find the relationship between

$$(1+\gamma k)(k+(M-1)m+M)+(1-\gamma)mM(k+1)$$

and

$$\gamma k(k+(M-1)m+M)+(1-\gamma)mM(k+1)$$

because we want to get the format

$$p_k = \square \frac{\gamma(k-1)(k-1+(M-1)m+M)+(1-\gamma)mMk}{\gamma k(k+(M-1)m+M)+(1-\gamma)mM(k+1)} \frac{k+(M-1)m+M}{k-1+(M-1)m+M} p_{k-1} \tag{B.2}$$

where

$$\begin{aligned} \square &= \frac{\gamma k(k+(M-1)m+M)+(1-\gamma)mM(k+1)}{(1+\gamma k)(k+(M-1)m+M)+(1-\gamma)mM(k+1)} \\ &= \frac{\gamma k^2+(Mm+M\gamma-m\gamma)k+Mm-Mm\gamma}{\gamma k^2+(Mm+M\gamma-m\gamma+1)k+M-m+2Mm-Mm\gamma} \end{aligned} \tag{B.3}$$

Factor the expression we have

$$\begin{aligned} &\gamma k^2+(Mm+M\gamma-m\gamma)q+Mm-Mm\gamma \\ &= \gamma(k+\frac{Mm+M\gamma-m\gamma+\sqrt{\triangle}}{2\gamma})(k+\frac{Mm+M\gamma-m\gamma-\sqrt{\triangle}}{2\gamma}) \end{aligned} \tag{B.4}$$

where

$$\triangle = M^2m^2+2M^2m\gamma+M^2\gamma^2-2Mm^2\gamma+2Mm\gamma^2-4Mm\gamma+m^2\gamma^2$$

And

$$\begin{aligned} &\gamma k^2+(Mm+M\gamma-m\gamma+1)k+M-m+2Mm-Mm\gamma \\ &= \gamma(k+\frac{Mm+M\gamma-m\gamma+\sqrt{\triangledown}+1}{2\gamma})(k+\frac{Mm+M\gamma-m\gamma-\sqrt{\triangledown}+1}{2\gamma}) \end{aligned} \tag{B.5}$$

where

$$\triangledown = M^2m^2+2M^2m\gamma+M^2\gamma^2-2Mm^2\gamma+2Mm\gamma^2-8Mm\gamma+2Mm-2M\gamma+m^2\gamma^2+2m\gamma+1$$

By iteration, we have

$$\begin{aligned} p_k &= \blacksquare \frac{(1-\gamma)mM}{\gamma k(k+(M-1)m+M)+(1-\gamma)mM(k+1)} \frac{k+(M-1)m+M}{(M-1)m+M} p_0 \\ &= \blacksquare \frac{1+(M-1)m+M}{\gamma k(k+(M-1)m+M)+(1-\gamma)mM(k+1)} \frac{k+(M-1)m+M}{(M-1)m+M} \end{aligned} \tag{B.6}$$

where

$$\begin{aligned} \blacksquare &= \prod_{i=1}^{k} \frac{(i+\frac{Mm+M\gamma-m\gamma+\sqrt{\triangle}}{2\gamma})(i+\frac{Mm+M\gamma-m\gamma-\sqrt{\triangle}}{2\gamma})}{(i+\frac{Mm+M\gamma-m\gamma+\sqrt{\triangledown}+1}{2\gamma})(i+\frac{Mm+M\gamma-m\gamma-\sqrt{\triangledown}+1}{2\gamma})} \\ &= \frac{B(k,\frac{Mm+M\gamma-m\gamma+\sqrt{\triangledown}+1}{2\gamma}+1)B(k,\frac{Mm+M\gamma-m\gamma-\sqrt{\triangledown}+1}{2\gamma}+1)}{B(k,\frac{Mm+M\gamma-m\gamma+\sqrt{\triangle}}{2\gamma}+1)B(k,\frac{Mm+M\gamma-m\gamma-\sqrt{\triangle}}{2\gamma}+1)} \end{aligned} \tag{B.7}$$

We use

$$\frac{\Gamma(x+n)}{\Gamma(x)} = (x+n-1)(x+n-2)\cdots x$$

and

$$B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

to derive Eq. (B.6).

References

[1] R. Albert, H. Jeong, A.-L. Barabási, Internet: Diameter of the world-wide web, nature 401 (6749) (1999) 130.

[2] S. Dorogovtsev, J. Mendes, Evolution of networks: from biological nets to the Internet and WWW, Oxford University Press, 2003.

[3] S. Li, C. M. Armstrong, N. Bertin, H. Ge, S. Milstein, M. Boxem, P.-O. Vidalain, J.-D. J. Han, A. Chesneau, T. Hao, et al., A map of the interactome network of the metazoan c. elegans, Science 303 (5657) (2004) 540–543.

[4] J. Leskovec, J. Kleinberg, C. Faloutsos, Graph evolution: Densification and shrinking diameters, ACM Transactions on Knowledge Discovery from Data (TKDD) 1 (1) (2007) 2.

[5] M. Newman, Networks: An Introduction, Oxford University Press, 2010.

[6] A.-L. Barabási, et al., Network science, Cambridge university press, 2016.

[7] D. J. Watts, S. H. Strogatz, Collective dynamics of ‘small-world’networks, nature 393 (6684) (1998) 440.

[8] A.-L. Barabási, R. Albert, Emergence of Scaling in Random Networks, Science 286 (5439) (1999) 509. doi:10.1126/science.286.5439.509.

[9] P. Erdős, A. Rényi, On the evolution of random graphs, Publ. Math. Inst. Hung. Acad. Sci 5 (1) (1960) 17–60.

[10] X. Li, G. Chen, A local-world evolving network model, Physica A: Statistical Mechanics and its Applications 328 (1-2) (2003) 274–286.

[11] A. Bonato, N. Hadi, P. Horn, P. Prałat, C. Wang, Models of online social networks, Internet Mathematics 6 (3) (2009) 285–313. doi:10.1080/15427951.2009.10390642.

[12] Z.-G. Shao, T. Chen, B.-q. Ai, Growing networks with temporal effect and mixed attachment mechanisms, Physica A: Statistical Mechanics and its Applications 413 (2014) 147–152.

[13] T. Carletti, F. Gargiulo, R. Lambiotte, Preferential attachment with partial information, The European Physical Journal B 88 (1) (2015) 18.

[14] P. K. Pandey, B. Adhikari, Context dependent preferential attachment model for complex networks, Physica A: Statistical Mechanics and its Applications 436 (2015) 499–508.

[15] P. K. Pandey, B. Adhikari, A parametric model approach for structural reconstruction of scale-free networks, IEEE Transactions on Knowledge and Data Engineering 29 (10) (2017) 2072–2085.

[16] S. Chattopadhyay, C. Murthy, Generation of power-law networks by employing various attachment schemes: Structural properties emulating real world networks, Information Sciences 397 (2017) 219–242.

[17] A.-L. Barabási, R. Albert, H. Jeong, Mean-field theory for scale-free random networks, Physica A: Statistical Mechanics and its Applications 272 (1-2) (1999) 173–187.

[18] S. Mossa, M. Barthélémy, H. Eugene Stanley, L. A. Nunes Amaral, Truncation of power law behavior in "scale-free" network models due to information filtering, Phys. Rev. Lett. 88 (2002) 138701. doi:10.1103/PhysRevLett.88.138701.
URL `https://link.aps.org/doi/10.1103/PhysRevLett.88.138701`

[19] H. Štefančić, V. Zlatić, Preferential attachment with information filtering—node degree probability distribution properties, Physica A: Statistical Mechanics and its Applications 350 (2-4) (2005) 657–670.

[20] J. M. Kleinberg, R. Kumar, P. Raghavan, S. Rajagopalan, A. S. Tomkins, The web as a graph: measurements, models, and methods, in: International Computing and Combinatorics Conference, Springer, 1999, pp. 1–17.

[21] A. Clauset, C. R. Shalizi, M. E. Newman, Power-law distributions in empirical data, SIAM review 51 (4) (2009) 661–703.

[22] A. Hagberg, P. Swart, D. S Chult, Exploring network structure, dynamics, and function using networkx, Tech. rep., Los Alamos National Lab.(LANL), Los Alamos, NM (United States) (2008).

[23] L. d. F. Costa, F. A. Rodrigues, G. Travieso, P. R. Villas Boas, Characterization of complex networks: A survey of measurements, Advances in physics 56 (1) (2007) 167–242.

[24] P. Holme, B. J. Kim, Growing scale-free networks with tunable clustering, Physical review E 65 (2) (2002) 026107.

[25] P. Van Mieghem, J. Omic, R. Kooij, Virus spread in networks, IEEE/ACM Transactions on Networking (TON) 17 (1) (2009) 1–14.

[26] M. E. Newman, Mixing patterns in networks, Physical Review E 67 (2) (2003) 026126.